Benchmark: sensitivity analysis of a flexible five-bar mechanism

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Abstract

The present document describes the flexible five-bar mechanism and the simulation chosen as a benchmark problem for sensitivity analysis of flexible multibody systems.

1 Description of the five-bar mechanism

The case study chosen is the five-bar mechanism composed of 4 rigid bars (one of them fixed) and a flexible bar (marked in green in figure 1).

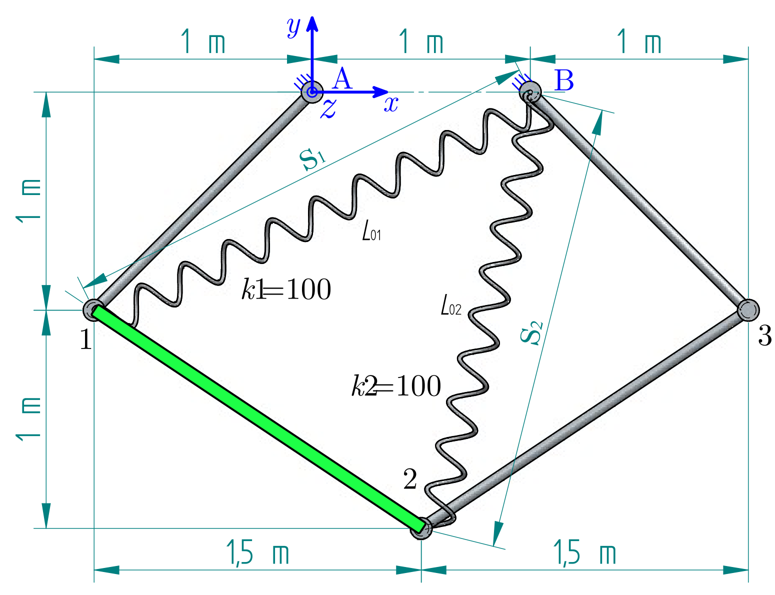

Figure 1: The five-bar mechanism

The multibody model and the dynamics of the mechanism under the action of gravity and spring forces are explained in the following benchmark link:

`https://www.iftomm-multibody.org/benchmark/problem/Dynamics_of_a_flexible_five-bar_mechanism/`

2 Five-bar mechanism: sensitivity problem description and solution

For the sensitivity analysis, the following array of objective functions is considered:

$$\boldsymbol{\psi} = \begin{bmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \end{bmatrix} \tag{1}$$

with

$$\psi^1 = \int_{t_0}^{t_F} \left(\mathbf{r}_2 - \mathbf{r}_{20}\right)^{\mathrm{T}} \left(\mathbf{r}_2 - \mathbf{r}_{20}\right) \mathrm{d}t = \int_{t_0}^{t_F} g_1 \mathrm{d}t \tag{2a}$$

$$\psi^2 = \int_{t_0}^{t_F} \dot{\mathbf{r}}_2^{\mathrm{T}} \dot{\mathbf{r}}_2 \mathrm{d}t = \int_{t_0}^{t_F} g_2 \mathrm{d}t \tag{2b}$$

$$\psi^3 = \int_{t_0}^{t_F} \ddot{\mathbf{r}}_2^{\mathrm{T}} \ddot{\mathbf{r}}_2 \mathrm{d}t = \int_{t_0}^{t_F} g_3 \mathrm{d}t \tag{2c}$$

where $\mathbf{r}_{20} = \begin{bmatrix} 0.5 & -2.0 & 0.0 \end{bmatrix}$ represents the initial position of point 2.

Three different sets of sensitivity parameters are considered:

1. Parameters affecting the forces: the natural lengths of the springs, $\boldsymbol{\rho}_1 = \begin{bmatrix} L_{01} & L_{02} \end{bmatrix}^{\mathrm{T}}$.

2. Parameters affecting the distribution of masses in rigid bodies: the mass of bar 1 and the longitudinal local coordinate of its center of mass, $\boldsymbol{\rho}_2 = \begin{bmatrix} m_{A1} & \bar{x}_{A1}^{G} \end{bmatrix}^{\mathrm{T}}$.

3. Parameters affecting kinematics of the rigid body: the length of bar A1, $\boldsymbol{\rho}_3 = \begin{bmatrix} L_{A1} \end{bmatrix}$.

4. Parameters affecting the flexible body geometry: the length of bar 12, $\boldsymbol{\rho}_4 = \begin{bmatrix} L_{12} \end{bmatrix}$. It should be pointed out that points 1 and 2 are considered to be in the tip faces of the flexible bar.

Then, the vector of parameters for the sensitivity analysis is:

$$\boldsymbol{\rho}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{\rho}_1^{\mathrm{T}} & \boldsymbol{\rho}_2^{\mathrm{T}} & \boldsymbol{\rho}_3^{\mathrm{T}} & \boldsymbol{\rho}_4^{\mathrm{T}} \end{bmatrix} \tag{3}$$

The outcome of the sensitivity analysis is the gradient of the objective function (2), $\boldsymbol{\psi}' = \frac{\mathrm{d}\boldsymbol{\psi}}{\mathrm{d}\boldsymbol{\rho}}$.

The initial sensitivities of the coordinates of points 1, 2 and 3 are the following:

$$\begin{bmatrix} r'_{1x} \\ r'_{1y} \\ r'_{2x} \\ r'_{2y} \\ r'_{3x} \\ r'_{3y} \end{bmatrix}_{t_0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -0.707\,106\,8 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.707\,106\,8 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.707\,106\,8 & 0.832\,050\,3 \\ 0 & 0 & 0 & 0 & 0 & -0.707\,106\,8 & -0.554\,700\,2 \\ 0 & 0 & 0 & 0 & 0 & -0.707\,106\,8 & 0.277\,350\,1 \\ 0 & 0 & 0 & 0 & 0 & -0.707\,106\,8 & 0.277\,350\,1 \end{bmatrix}_{t_0} \tag{4}$$

The reference solutions provided for the dynamics and sensitivity analysis have been obtained with a time step of 5×10^{-5} s with two different floating frame of reference sensitivity formulations, with the flexibility of bar 12 parameterized by means of 30 modes of deformation coming from a modal analysis. The modes of deformation have been calculated from a modal analysis with the tip faces behaving as rigid elements with Y translation and X and Z rotations blocked.

The objective functions and their sensitivities are shown in figures 2 and 3 and the results are also included in the data files *Obj1Grad.csv*, *Obj2Grad.csv* and *Obj3Grad.csv*.

3 Benchmark error calculation

The final values of the sensitivities ($t = 5$ s) for the reference solution are included in table 1 along with the maximum allowed error for each sensitivity.

The allowed error is 0.1% of the amplitude of each reference gradient component along time. The amplitude is measured as the maximum value minus the minimum value of the reference results included in files `Obj1Grad.csv`, `Obj1Grad.csv` and `Obj1Grad.csv` (first column is the time step). Therefore:

$$error^{i,j} = \frac{1}{A^i_j}\left(\left(\boldsymbol{\psi}^i\right)_{\rho_j} - \left(\boldsymbol{\psi}^i\right)^{Ref}_{\rho_j}\right)^{t_F} \tag{5}$$

Results are considered as acceptable if the summation of errors per gradient component is lower than 0.6%, thus:

$$error = \sum_{i=1}^{3}\sum_{j=1}^{6} error^{i,j} = 0.006 \tag{6}$$

References

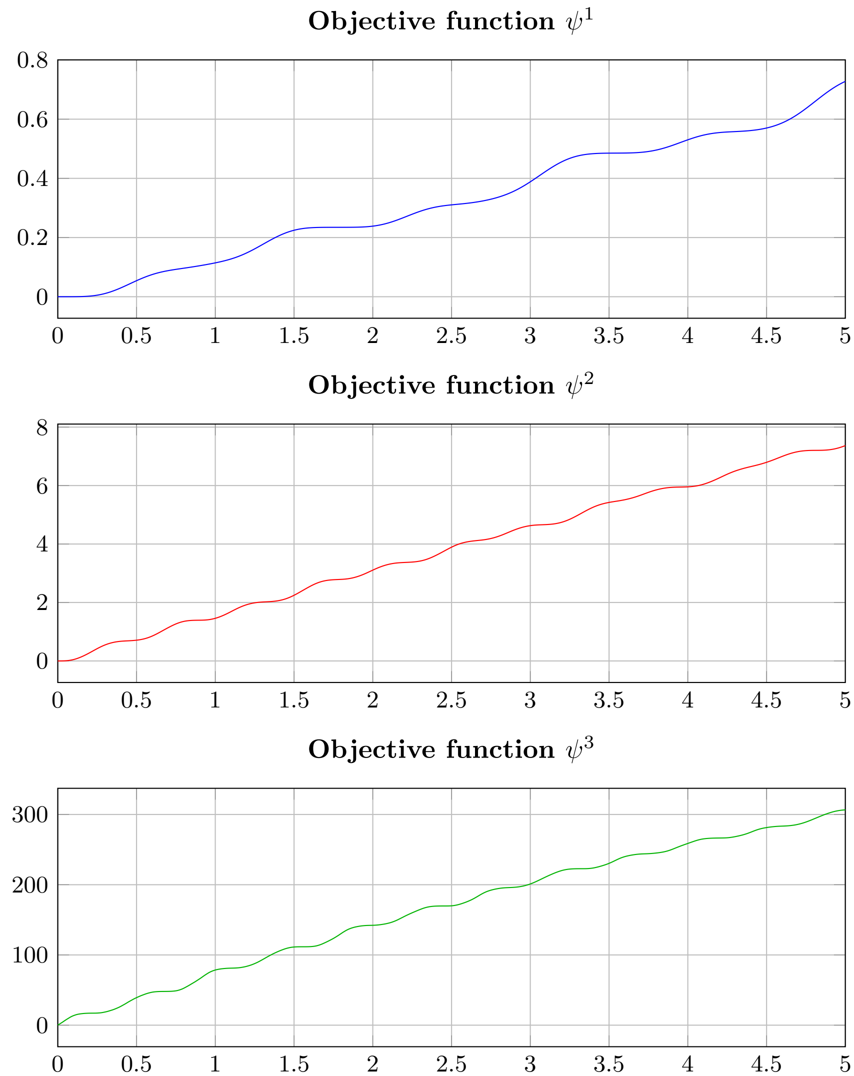

Figure 2: Objective functions over time $\int_{t_0}^{t} g_i \mathrm{d}t$ with $t \in [0, t_F]$, $i = 1, 2, 3$.

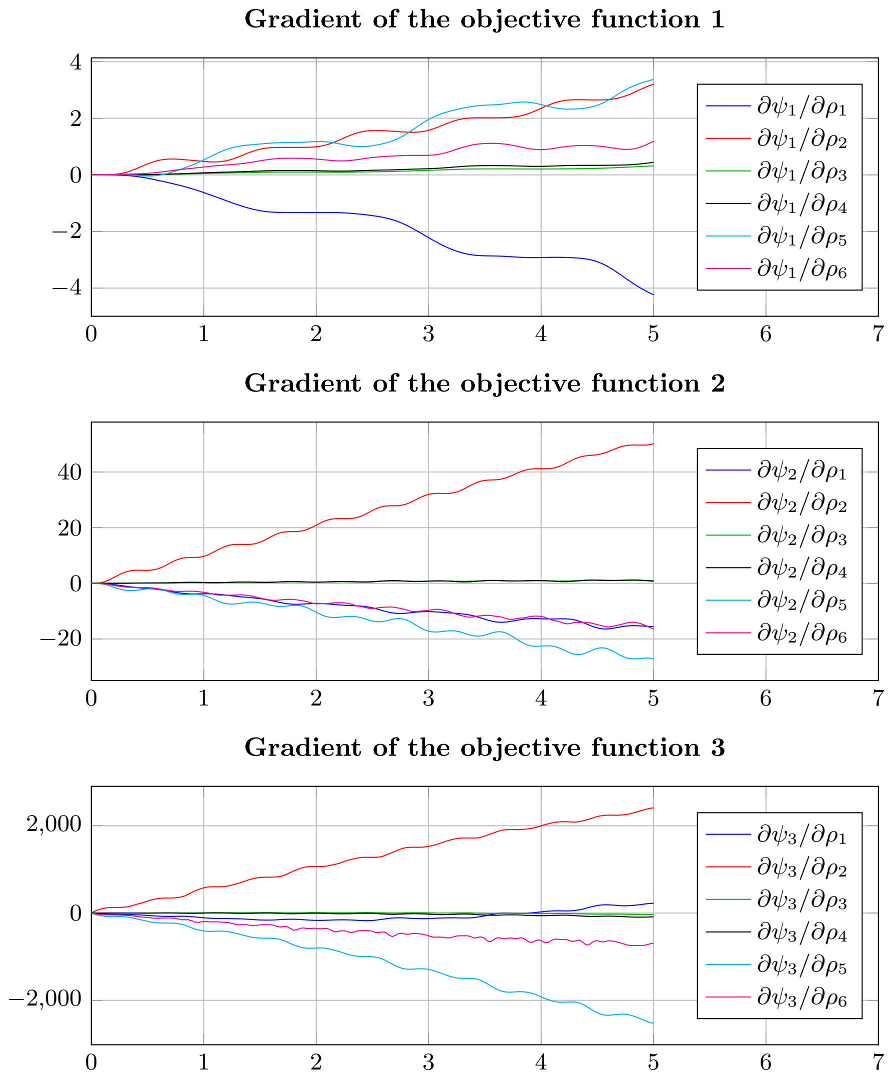

Figure 3: Gradients of the objective functions with respect to the design parameters.

	Reference solution	Amplitude
$\left(\psi^1\right)'_{L_{s1}}$	$-4.234\,799$	$4.234\,799$
$\left(\psi^1\right)'_{L_{s2}}$	$3.201\,432$	$3.201\,432$
$\left(\psi^1\right)'_{m_{A1}}$	$3.190\,141 \times 10^{-1}$	$3.190\,141 \times 10^{-1}$
$\left(\psi^1\right)'_{x^G_{A1}}$	$4.425\,693$	4.425696e-01
$\left(\psi^1\right)'_{L_{A1}}$	$3.374\,414$	$3.414\,335$
$\left(\psi^1\right)'_{L_{12}}$	$1.187\,308$	$1.187\,308$
$\left(\psi^2\right)'_{L_{s1}}$	$-1.550\,864 \times 10^{1}$	$1.639\,105 \times 10^{1}$
$\left(\psi^2\right)'_{L_{s2}}$	$5.019\,048 \times 10^{1}$	$5.019\,048 \times 10^{1}$
$\left(\psi^2\right)'_{m_{A1}}$	$9.698\,642 \times 10^{-1}$	$1.167\,627$
$\left(\psi^2\right)'_{x^G_{A1}}$	$7.431\,987 \times 10^{-1}$	$1.201\,000$
$\left(\psi^2\right)'_{L_{A1}}$	$-2.717\,809 \times 10^{1}$	$2.717\,809 \times 10^{1}$
$\left(\psi^2\right)'_{L_{12}}$	$-1.637\,327 \times 10^{1}$	$16.373\,27 \times 10^{1}$
$\left(\psi^3\right)'_{L_{s1}}$	$2.234\,345 \times 10^{2}$	$3.982\,183 \times 10^{2}$
$\left(\psi^3\right)'_{L_{s2}}$	$2.409\,527 \times 10^{3}$	$2.409\,527 \times 10^{3}$
$\left(\psi^3\right)'_{m_{A1}}$	$-3.294\,194 \times 10^{1}$	$4.412\,995 \times 10^{1}$
$\left(\psi^3\right)'_{x^G_{A1}}$	$-8.649\,282 \times 10^{1}$	$9.747\,709 \times 10^{1}$
$\left(\psi^3\right)'_{L_{A1}}$	$-2.524\,507 \times 10^{3}$	$2.524\,507 \times 10^{3}$
$\left(\psi^3\right)'_{L_{12}}$	$-6.903\,434 \times 10^{2}$	$7.443\,608 \times 10^{2}$

Table 1: Reference solution for sensitivities at $t = 5$ s. and maximum allowed error.